REVIEWS

The Structure of Turbulent Shear Flow, by A. A. TOWNSEND. Cambridge University Press, 1956. 315 pp. 40*s*. or \$7.50.

Any steady flow of a fluid is unstable when the Reynolds number (product of a typical flow velocity with a typical length scale, divided by the kinematic viscosity) is great enough, although the required magnitude varies extremely widely with the type of flow and the size of such disturbances as are present. At Reynolds numbers just large enough to produce instability, the disturbed flow may settle into a regular eddy pattern (such as the Kármán trail or Bénard cells), but at higher values an irregular, 'turbulent' flow is produced, in which the velocity at any point varies in a statistically random manner Fourier analysis shows that fluctuations in a range of frequencies are present, the width of the range increasing with Reynolds number.

The study of turbulent flow is a part of science not founded on the 'principle of uniformity' as expounded by students of scientific method. The velocity record at a fixed point obtained by a hot-wire anemometer recording on to an oscilloscope is quite different every time a given flow (under fixed conditions) takes place. The aim, throughout the seventy years since Reynolds initiated the study of turbulence, has been to recover some partial degree of uniformity by statistical studies, that is, studies of averages of various quantities. In theory, a fluid flow which is known precisely at one instant can have its later development predicted uniquely. Hence, if a complete statistical knowledge of the flows generated in a certain way were available for one instant of time, such knowledge could in theory be deduced for later instants. However, it is of the essence of the problem that the statistical distribution of the initial random disturbances that lead to turbulence is not known in practice. Hence, no definite results can be expected unless there is a tendency for the development with time to obliterate special effects of this initial distribution. Actually, some such tendency is observed, but it is only partial.

Other branches of statistical physics have been more successful than turbulence theory. The obliteration tendency appears most strongly in the kinetic theory of gases, but the 'molecular chaos', which implies a statistical independence of velocity distributions at different places however close, makes any analogy with turbulence tenuous indeed. The existence of spatial correlation brings turbulence closer to that other rather unsuccessful subject, the kinetic theory of liquids.

Again, there is an obvious difference from linear problems where response is frequency-conserving. A better analogy is provided by forced mechanical vibrations whose amplitude is governed by non-linear forces which transfer energy between modes, such as could be observed in the film of the Tacoma Bridge disaster. However, turbulence at high Reynolds number is different in that it possesses a far greater range of wave-numbers, due to weaker internal damping.

This presence of modes of fluctuation with a wide range of wave-numbers, transferring energy among themselves as a result of the non-linear character of the inertial forces, is the basis of the Kolmogoroff–Onsager theory of the 'small eddies', that is, of the components of flow which in a spatial Fourier analysis have large wave-number. This theory is based on the idea that transfer of energy takes place in the direction of increasing wave-number, and in fairly short steps (in terms of the ratio of wave-number of a larger eddy to that of the smaller eddy which draws off some of its energy). Our knowledge of the basic instability of the steady flow already indicates that energy passes from that minimum-wave-number motion most readily to a motion with wave-number greater by half to one order of magnitude, and if this is true right up the wave-number scale the idea follows that the small eddies may have been reached by a statistical combination of many elementary processes which could obliterate the pecularities of the original large-scale disturbances. Hence they may be in an equilibrium which depends only on the viscosity which dissipates their energy and on the total rate at which energy is passed down to them, or (in the case of flows where this rate is changing with time) in a moving equilibrium which may depend also on the rate of change of that rate.

This theory has given a fairly satisfactory statistical picture of the small eddies, close to isotropy and equilibrium, which is useful in many applications. However, because of the increase in rate of energy loss to smaller eddies and in rate of dissipation as wave-number increases, the actual energy in the small eddies at any instant is a small fraction of the whole. The processes, if any, that govern the character of the 'energy-containing eddies' (those larger ones which contain the bulk of the turbulent energy) have remained a mystery, except that some progress has been made in the case of homogeneous turbulence (with a uniform mean flow), which was reviewed in 1953 by Dr G. K. Batchelor in the Cambridge series of Monographs on Mechanics and Applied Mathematics (this book contains also a good account of the small eddies).

Now at last, in the same series, a book has come out which, largely eschewing the small eddies as in the main already understood, takes as its principal subject the problem of how, in general turbulent flows, the character of the energy-containing eddies is controlled. It is called *The Structure of Turbulent Shear Flow*, presumably to disavow any restriction to the cases with uniform mean flow already described by Dr Batchelor, and perhaps also to exclude turbulent flows due to thermal convection, but it might have been called simply *Turbulence*. Its author, Dr Alan Townsend, has a combination of experimental and theoretical ability which no-one entering the subject has possessed since Sir Geoffrey Taylor (in whose Cambridge laboratory he works, and whose encouragement he acknowledges in a preface), and which were almost certainly essential requisites for his task. He has been developing his ideas in a number of exciting articles over the last nine years, but it has not been easy to follow from these what the conclusions would be, as in many cases the views expressed in one article were retracted in a succeeding one, and the author was plainly feeling his way. But now we have an opportunity to take in as a whole the theory and the experimental evidence on which it is based, both being very plainly set out in the book under review.

A number of basic ideas dominate Dr Townsend's account of an extensive range of different flows. The first three are simple general ideas which have been adopted fairly widely before, although not always, as Dr Townsend points out, with a full understanding of which results follow from them and which from more specialized assumptions about structure. They say that at sufficiently high Reynolds number the energy-containing eddies (and, generally, all except the smallest-scale features of the flow) (i) are independent of Reynolds number (because the viscosity affects only the small eddies) and in time reach a state in which (ii) their velocity and length-scales may be changing but other quantities of the same dimensions change in proportion, and which (iii) is independent of the details of such boundary conditions as are basically responsible for the turbulence, depending only on an overall resultant of them. The last principle is close to that of St Venant in elasticity, and as in that subject it is the resultant force (together with, sometimes, a length scale) which matters. Thus, turbulence in a jet depends on its net thrust, in a wake on the drag of the body producing it, and in pipe flow on the wall resistance. The independence of Reynolds number (principle (i)) and of details of geometry is postulated only for given values of these forces, which on the other hand for a given geometry may vary with Reynolds number owing to the kinematic boundary condition of no slip being applied in the 'viscous layer' near the wall where the variation of mean velocity has a very small length-scale.

Dr Townsend begins his book with two chapters in which he sets forth the principal experimental and mathematical tools which are available for the study of turbulence; here, he departs from previous practice principally by stressing the idea of an eddy as something finite in extent (and even showing a preference for it being round, as indeed so many eddies observed in nature are!) rather than as being simply a Fourier component. In chapter 3 he sets out the evidence for the three principles just outlined in the case of turbulence with uniform mean flow, for which particularly detailed evidence is available. If the principles were completely false the subject of the book could hardly be said to exist, in that practically no uniformity could be imposed on the data. In fact they are broadly true, although only to a rough approximation.

He then develops a number of further principles, and devotes the rest of the book to showing how they illuminate the main features of turbulent flow in wakes, jets, mixing zones, pipes, channels, boundary layers, and between rotating cylinders, and to showing how well the various principles are borne out by experimental observation.

One principle that could be expressed better than in the book itself is that when turbulence is subject to a constant mean rate of strain the anisotropy of its energy tensor tends to a limiting form. If

$$S_{ij} = \tfrac{1}{2}(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$$

represents the mean rate of strain and $\overline{u_i u_j}$ the turbulent energy tensor, this anisotropy is represented by the behaviour of $\overline{u_i u_j}/\overline{q^2}$ (where $\overline{q^2} = \overline{u_i u_i}$). This must depend on S_{ij} in a manner invariant under change of axes, and hence it can only be of the form

$$A\delta_{ij} + BS_{ij} + CS_{ik}S_{kj}, \tag{1}$$

where A, B, C are functions of the invariants $S_{kl}S_{kl}$ and Δ (the determinant of S_{ij}). A form which fits the experimental observations cited by Dr Townsend is

$$\frac{\overline{u_i u_j}}{\overline{q^2}} = \tfrac{1}{3}\delta_{ij} - \frac{0{\cdot}2S_{ij}}{(S_{kl}S_{kl})^{1/2}}. \tag{2}$$

This seems more satisfactory than Dr Townsend's form, which is not invariant. The reason why C in (1) is put equal to zero is that in plane strain (in the $x_1 = 0$ plane) the value of Dr Townsend's $K_2 = 3\overline{u_1^2}/\overline{q^2} - 1$ is observed to vanish. The best-known case of equation (2) is the tendency of the 'shear coefficient' $(-\overline{uv}/\overline{u^2})$ to be about 0·4 in simple shearing flow with $dU/dy > 0$.

The physical explanation of the principle just described is based on the one hand on the known tendency for strain in one direction to increase vorticity in that direction and hence to favour velocity components in the direction at right angles (at the expense of components in that direction). To balance this tendency we no longer have the once-credited 'general tendency to isotropy' of the energy-containing eddies, which this book (together with a number of other contemporary investigations) finally explodes. If, in fact, the mean rate-of-strain is removed, the anisotropy is found to remain practically unchanged (until some new mean rate-of-strain is applied). In place of this, we have the suggestion that eddies too much stretched in the direction of mean strain decay more rapidly than less elongated ones.

A second important principle inferred by Dr Townsend from the data takes its simplest form in free turbulence. This consists of a region of 'uniformly turbulent' fluid (containing all the vorticity and practically all the turbulent energy, uniformly distributed, but with anisotropy as governed by the former principle) with a bounding surface of extremely irregular shape, different at different instants. The uniformity of the turbulent fluid is explained as due to diffusion. The spread of the turbulent region is explained as due to contortion of the bounding surface by 'large eddies' and, on a smaller scale, by all the eddies, which taken together produce an extremely large boundary area over which viscous forces can diffuse vorticity. That this process operates very effectively at high Reynolds numbers is clear from the fact that the observed rate of growth does not decrease as the Reynolds number increases (that is, as the time scale for operation of viscous forces increases relative to the basic kinematic time scales).

The 'large' eddies (larger even that those which contain the bulk of the turbulent energy control the ultimate speed of spread of the turbulence.

They derive energy from the primary instability of the mean flow and pass energy down to the energy-containing eddies. Here Dr Townsend assumes (with some experimental support) that one of the simplest kinds of eddy capable of extracting energy from the mean flow predominates in this range of sizes. This ends up as a round eddy (with axis in the direction of flow) which just fills up the region of substantial mean rate-of-strain of any one sign. Before the eddy is sheared into this form it is bent back at an angle and during this shearing process it gains energy (as the area of its closed streamlines decreases).

Dr Townsend derives the energy of the main turbulence by applying a condition that it extracts energy from a 'large eddy' of this form at the same rate at which the large eddy extracts energy from the mean flow. Thus, he is applying arguments similar to the 'equilibrium' arguments of Kolmogoroff at the small-wave-number end of the scale. The rate of dissipation of energy of the large eddy by turbulence is computed by the use of an 'eddy viscosity', and it is the magnitude of this eddy viscosity (determined empirically in the older theories) which emerges from the Townsend theory as a predicted quantity in satisfactory agreement with experiment.

The mere existence of an effective eddy viscosity is not, however, taken as an *a priori* assumption. Dr Townsend shows how it follows from his theory of the effect of mean rate-of-strain on anisotropy. One of his 'large eddies' effectively re-orients the local rate-of-strain environment of the turbulence, and this alters the local anisotropy of the energy-containing eddies and so produces an addition to the Reynolds stresses. The derivation is simpler from equation (2) above that from Townsend's form, and so may as well be given. A change δS_{ij} in the mean rate of strain produces a change

$$\delta(-\overline{u_i u_j}) = 0{\cdot}2\overline{q^2}\left\{\frac{\delta S_{ij}}{(S_{kl}S_{kl})^{1/2}} - \frac{S_{ij}S_{kl}\delta S_{kl}}{(S_{kl}S_{kl})^{3/2}}\right\} \tag{3}$$

in the Reynolds stresses. In the case of a plane shearing motion only the term $S_{12} = S_{21} = \frac{1}{2}dU/dy$ is present in S_{ij}, and then (3) becomes

$$\frac{0{\cdot}2\overline{q^2}\sqrt{2}}{dU/dy}\begin{vmatrix} \delta S_{11} & 0 & \delta S_{13} \\ 0 & \delta S_{22} & \delta S_{23} \\ \delta S_{13} & \delta S_{23} & \delta S_{33} \end{vmatrix}, \tag{4}$$

which, for an eddy without an especially large δS_{12} component of rate-of-strain, represents closely a viscous resistance by an effective viscosity equal to

$$-\frac{0{\cdot}2\overline{q^2}}{(\sqrt{2})dU/dy} = -\frac{0{\cdot}4\overline{u^2}}{dU/dy} = -\frac{\overline{uv}}{dU/dy}, \tag{5}$$

which is the ordinary 'eddy viscosity' of the old theories.

Throughout the book Dr Townsend walks a very thin tight-rope in his attitude to eddy viscosity. He expects the reader to believe that it is 'accidental' (p. 161) that the assumption of an eddy viscosity in the ordinary sense, which at any cross-section of the turbulent layer is constant within the 'turbulent fluid' and zero outside it, gives dead-on agreement with the observed mean velocity for all free turbulent flows. Earlier (p. 44), he admits the success with which Heisenberg and his followers applied the eddy viscosity idea to predict the whole spectrum of the energy-containing eddies in isotropic turbulence, but presumably he regards this as accidental, too! The reviewer would agree that Dr Townsend's data show that there is no simple single reason why eddy viscosity seems to work, but rather a number of conspiring circumstances—for example, there is Dr Townsend's point that where dU/dy is nearly zero, the eddies consist partly of those which have been positively sheared and partly of those which have been negatively sheared, and hence $\overline{uv}$ also will be nearly zero. However, the rather sweeping and undeveloped arguments on which the older ideas are rejected in this book (for example, on p. 94) do not greatly reinforce the simple objection (p. 128) that they contain undetermined constants.

Chapters 4 to 8 are devoted to the development of the principles described above and their application to flow in wakes and jets. In these applications, it is clear that something has been sacrificed by the decision to study only flows which have reached the self-preserving state (principle (ii)), because the time taken by turbulence to change from one type to another is in fact long. In particular, the strong 'mixing-region' turbulence takes a long time to settle down into the relatively weaker wake turbulence or jet turbulence. On the other hand, the success of this book in throwing light on turbulence completely vindicates Dr Townsend's decision to confine his observations at this stage to the regions of self-preserving flow, far downstream, where this settling-down has taken place, and where there was more hope of uncovering the mechanics of the motion.

In chapter 9, on flow in pipes and channels, the complications due to the presence of a wall are first introduced. Correlation and other measurements in the region of early constant stress near the wall are shown to be consistent with the idea that cylindrical eddies like the 'large eddies' of free turbulent flow sit on the wall and extract energy from the mean flow at the same rate as they pass energy down to not-so-large eddies. This leads to the right ratio of eddy viscosity to distance from the wall.

In chapters 10 and 11 the ideas appropriate to free turbulent flow and to flow near a wall are combined in a discussion of the turbulent boundary layer. The theory of equilibrium of the large eddies is still remarkably effective and the evidence from correlation measurements for their presence and general character is good. In chapter 11 the ideas of the book (including, it must be noted, an eddy viscosity!) are used effectively on the interesting self-preserving flows with adverse pressure gradient recently discovered by Dr Clauser. In both chapters a discussion of three-dimensional effects including secondary flows is given. Finally, in chapter 12, the flows between rotating cylinders are fitted to some extent into the general framework of the book.

A large measure of sound understanding, with some notable gaps, was already present in the world literature on turbulent shear flow, so far as the behaviour of the mean flow and the small eddies was concerned. Dr Townsend's book, while not omitting this material, will always be remembered principally for having first made a reasonably convincing bridge between these extreme features of the flow and given us a pattern for the intermediate parts of the spectrum and a way of thinking quantitatively about the larger eddies that is sufficiently consistent with both the theoretical and the experimental evidence.

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